Markov Processes

- state space is continuous
- <u>time</u> is continuous

$$\int dx\, x^m \sum_{n=0}^{\infty} \left(-\frac{\partial}{\partial x}\right)^n \frac{\mu_n}{n!}\, \delta(x) = \sum_{n=0}^{\infty} \frac{\mu_n}{n!} \int dx\, x^m \left(-\frac{\partial}{\partial x}\right)^n \delta(x)$$

$$= \sum_{n=0}^{\infty} \frac{\mu_n}{n!} \underbrace{\int dx\, \delta(x) \left(+\frac{\partial}{\partial x}\right)^n x^m}_{m!\,\delta_{nm}} = \frac{\mu_m}{m!}\, m! = \mu_m$$

$$\frac{\partial}{\partial t} P(y,t|x_0,t_0) = \mathcal{L}\, P(y,t|x_0,t_0)$$

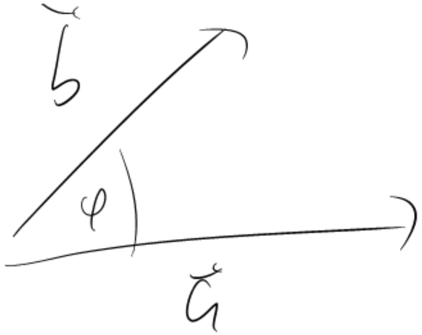
$$\mathcal{L} = \sum_{n=1}^{\infty} \left(-\frac{\partial}{\partial x}\right)^n D^{(n)}(x,t)$$

deterministic: $\dot{x} = f(x)$, $P(x,t|x_0,t_0) =$

$$= \delta\left[x - x(t)\right] \qquad x(t=t_0) = x_0$$

$$\frac{\partial}{\partial t} \delta\left[x - x(t)\right] = \frac{\partial}{\partial x} \delta\left[x - x(t)\right]\left[-\dot{x}(t)\right] = \qquad \mathcal{L}\,\delta(x - x(t))$$

$$= \frac{\partial}{\partial x} \delta\left(x - x(t)\right)\left[-f(x(t))\right] = -\frac{\partial}{\partial x} \delta\left(x - x(t)\right) f(x) \qquad \underline{\mathcal{L} = -\frac{\partial}{\partial x} f(x)}$$



$$(\vec a \cdot \vec b)^2 = a^2 b^2 \cos^2\varphi$$
$$\le a^2 b^2$$

$$\underline{\underline{\mu_{m+n}^2}} \le \mu_{2m}\, \mu_{2n}$$

$$D^{(n)} = \lim_{t\to 0} \frac{1}{n!\,t}\, \mu_n \qquad \mu_n = n!\,t\, D^{(n)}$$

$$\left[(m+n)!\,t\, D^{(m+n)}\right]^2 \le \left[(2m)!\,t\, D^{(2m)}\right]\left[(2n)!\,t\, D^{(2n)}\right]$$

$$\left(D^{(m+n)}\right)^2 \le \frac{(2m)!\,(2n)!}{\left((m+n)!\right)^2}\, D^{(2m)}\, D^{(2n)}$$

$$D^{(2N)} = D^{(2N+1)} = \dots = 0$$

$$\Rightarrow D^{(N+1)} = D^{(N+2)} = \dots = 0$$