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Director Gratings and Light Diffraction in the Nematic Cells with Spatially Modulated Easy Axis

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Periodic spatial modulation of easy axis with period $2\pi/Q$ onto the cell surfaces leads to the appearance of the director grating with the same period in the LC bulk. Incident light wave intensity modulated with period $2\pi/\Delta q$ induces additional director gratings in the cell bulk with periods $2\pi/\Delta q$, $2\pi/(Q+\Delta q)$, $2\pi/(Q-\Delta q)$. Diffraction of a probe light beam on such gratings is investigated for both planar and nearly homeotropic geometry of the director distribution and for strong and weak anchoring of director on the cell surfaces.

Keywords: Easy axis modulation; director gratings; light diffraction

1. INTRODUCTION

Interaction of nematic liquid crystal (NLC) with incident light field leads to director deviation from its initial state. In case of spatially modulated light-field intensity director gratings appear in the NLC bulk [1, 2, 3]. On the other hand, the orientation of director depends essentially on the boundary conditions on the cell surfaces and great attention has been focused on their control. Nowadays the mechanisms used for such a control are: exposure technique that governs the tilt angle [4], the oblique irradiation of photosensitive PVCN polymers by polarized UV-light which influences both the tilt angle and anchoring energy [5], heating treatment [6]. Recently, the new effect of NLC alignment has been reported, where the change of anchoring parameters occurs as a result of the light action on the bulk of a light-sensitive LC-azo dye mixture [7].

Thus, a recent possibility is to control the anisotropy in alignment layers, and thereby the anchoring parameters, by optical means. Since light can be focused to micrometre-sized pixels, this in principle enables varying the liquid crystal orientation with high spatial frequency, and enables applications such as high density optical data storage and the construction of complex electro-optic devices.

In this paper we assume that the direction of easy orientation axis is periodically modulated along one of the coordinate axes (similar boundary conditions were considered by G. Barbero *et al.* [8]) and obtain the distribution of director in the cell bulk in the spatially-modulated light wave field. Starting from this distribution and supposing that tested light beam does not influence the orientational field of director, we study diffraction of light on the appearing director grating. Finally, we consider the influence of finite anchoring of director on the boundaries.

2. FREE ENERGY OF THE CELL

We consider the NLC cell bounded at the planes $z = 0, L$. In the one-elastic-constant approximation its free energy in the light-wave field takes the form

$$F = \frac{K}{2}\int\{(\mathrm{div}\vec{n})^2 + (\mathrm{rot}\vec{n})^2\}dV - \frac{1}{16\pi}\int \varepsilon_{ij} E_i E_j^* dV + F_s,$$

$$F_s = -\frac{1}{2}W\int_s (\vec{n}\vec{e})^2 dS, \quad W > 0 \tag{1}$$

where the surface free energy F_s is taken in the form of Rapini potential [9], $\varepsilon_{ij} = \varepsilon_\perp + \varepsilon_a n_i n_j$ is the dielectric susceptibility tensor, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$. Note, that a unit vector $\vec{e}$ along the easy orientation axis can depend on the coordinates.

We assume that two plane monochromatic light waves are incident on the cell making the angle 2θ between their vectors (Fig. 1). The waves have equal amplitudes and are polarized along the same direction in the xz plane but their wave vectors are oriented symmetrically with respect to this plane

$$\vec{E}_{1,2} = \vec{E}_0 \exp i(\omega t - \vec{q}_{1,2}\vec{r}),\ \vec{E}_0 = E_0(1,0,0),\ \vec{q}_{1,2} = q(0, \pm\sin\theta, -\cos\theta). \tag{2}$$

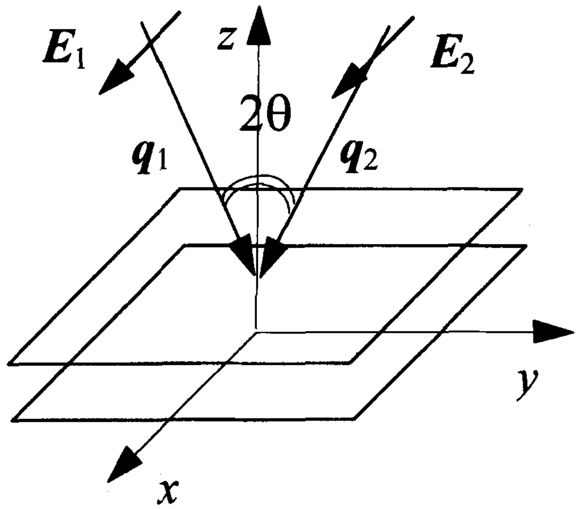

FIGURE 1 Considered geometry.

In consequence of interference in the cell volume the light wave field take the form

$$\vec{E} = \frac{1}{2}[\vec{E}(\vec{r})\exp(-i\omega t) + \vec{E}^*(\vec{r})\exp(i\omega t)], \tag{3}$$

where $\vec{E}(\vec{r}) = 2\vec{E}_0 \cos(1/2\Delta q y)\exp(iq\cos\theta z)$, $\Delta q = 2q\sin\theta$.

3. PERIODIC SPATIAL MODULATION OF PLANAR EASY AXIS

3.1. Director Distribution in the Light Field

Let the director of the NLC cell be parallel to the xy plane and the easy orientation axis on the cell surfaces be modulated with spatial period $2\pi/Q$ along the y axis. For the sake of simplicity we assume here the infinite anchoring of the director with the cell surfaces ($W = \infty$).

To obtain the light field in the cell we assume the polarization vectors of ordinary and extraordinary waves follows the local orientation of director. This assumption corresponds to the propagation of light in the Mauguin limit [11]. Defining the director as $\vec{n} = (\cos\varphi, \sin\varphi, 0)$ and introducing the vector $\vec{d} = (-\sin\varphi, \cos\varphi, 0) = \partial\vec{n}/\partial\varphi$ perpendicular to the director we can write down the field in the cell as

$$\vec{E} = \vec{E}_e + \vec{E}_o, \tag{4}$$

where

$$\vec{E}_e = (\vec{E} \cdot \vec{n})_{z=L} \vec{n}^* = E(z=L) \cos\varphi_L(y) \exp[i\,\omega/c\, n_e(L-z)]\,\vec{n},$$

$$\vec{E}_o = (\vec{E} \cdot \vec{d})_{z=L} \vec{d} = -E(z=L) \sin\varphi_L(y) \exp[i\,\omega/c\, n_o(L-z)]\,\vec{d}$$

Here $E(z=L)$ is defined by formula (3) for $E(\vec{r})$ putting here $z = L$, $\varphi_L(y)$ is the director angle on the plane $z = L$, n_e, n_o are extraordinary and ordinary refractive indexes correspondingly. Because of the phase difference $\omega/c(n_e - n_o)$, the field in the cell is in general elliptical and its polarization state changes along z coordinate. This leads to the periodic change of the torque of field origin and finally to the modulation of director distribution along z coordinate.

The director distribution $\vec{n}(y, z)$ in the expression (4) for the field in the cell is assumed to be already distored by the incident light wave, so the equation for director distribution in the cell is self-consistent.

After the minimization of the free energy (1) and substituion of the field (4) one can obtain the next equation for the director angle φ

$$L^2 \Delta_{yz} \varphi = I\{1 + \cos(\Delta q y)\} \sin 2\varphi_L(y) \cos k(L-z), \tag{5}$$

where $I = \varepsilon_a E_0^2/8\pi K$, $k = \omega/c(n_e - n_o)$.

Boundary conditions for the director take the form

$$\varphi|_{z=0} = \varphi_0(y) = \varphi_0 + \Delta\varphi_0 \sin Qy,$$

$$\varphi|_{z=L} = \varphi_L(y) = \varphi_L + \Delta\varphi_L \sin Qy. \tag{6}$$

in accordance with the periodic modulation of the easy axes along the y axis.

In the absence of the light field the equation (5) is homogeneous and its solution takes the form

$$\varphi^0(y, z) = \left(1 - \frac{z}{L}\right)\varphi_0 + \frac{z}{L}\varphi_L + \frac{\Delta\varphi_L \sinh Qz + \Delta\varphi_0 + \sinh Q(L-z)}{\sinh QL} \sin Qy. \tag{7}$$

Thus, the director grating with spatial period $\Lambda = 2\pi/Q$ appears in the cell bulk due to the periodic modulation of the easy axis orientation on the cell surfaces. For the small periods of easy axis modulation ($\Lambda \ll L$) the amplitude of grating is damped exponentially with the distance from the cell planes and the damping coefficient is defined by the ratio L/Λ. In the case of the large spatial periods ($\Lambda \gg L$) the director distribution (7) has the form

$$\varphi^0(y,z) = \left(1 - \frac{z}{L}\right)(\varphi_0 + \Delta\varphi_0 \sin Qy) + \frac{z}{L}(\varphi_L + \Delta\varphi_L \sin Qy), \tag{8}$$

that is the director grating with period Λ arises in the whole cell bulk.

In order to solve the inhomogeneous equation (5) let's put $\varphi(y,z) = \varphi^0(y,z) + \psi(y,z)$, where function $\psi(y,z)$ satisfies the equation (5) with zero boundary conditions $\psi|_{z=0,L} = 0$.

Assuming $\Delta\varphi_L \ll 1$ and solving the equation for $\psi(y,z)$ one can obtain the expression for the director angle $\varphi(y,z)$ in the form

$$\varphi = \varphi^0 + \varphi^1 + \varphi^2 + \varphi^3 + \varphi^4, \tag{9}$$

where

$$\varphi^1 = I\sin(2\varphi_L)\{(1-\cos kL)\tfrac{z}{L} + \cos kL - \cos k(L-z)\},$$

$$\varphi^2 = 2I\Delta\varphi_L Z(z,Q)\cos(2\varphi_L)\sin Qy,$$

$$\varphi^3 = I\sin 2\varphi_L Z(z,\Delta q)\cos(\Delta q y),$$

$$\varphi^4 = I\Delta\varphi_L \cos 2\varphi_L$$

$$\{Z(z,Q_1)\sin(Q_1 y) + Z(z,Q_2)\sin(Q_2 y)\},$$

$$Z(z,Q) = \frac{1}{L^2(k^2+Q^2)}\left\{\frac{\sinh Qz + \cos kL \sinh Q(L-z)}{\sinh QL} - \cos k(L-z)\right\},$$

$$Q_1 = Q + \Delta q, \; Q_2 = Q - \Delta q.$$

One can see that under the action of the light wave field (3) the initial director distribution φ^0 with the spatial period Λ is modified and the dynamic director gratings with periods $2\pi/\Delta q$, $2\pi/(Q+\Delta q)$, $2\pi/(Q-\Delta q)$ appear additionally in the NLC bulk. However, the amplitude of the director gratings induced by field is rather small. For typical parameters $L \sim 10$ μm, $n_e - n_o = 0.1$, $\lambda \sim 0.5$ μm the factor $kL \sim 10$, i.e. even for the light field intensity significantly exceeding the intensity of the light-induced Frederiks threshold in the cell ($I \sim \pi^2$), the deviations of director induced by incident light field can not be taken into account.

3.2. Diffraction on the Planar Director Grating

Suppose that the planar director grating has been created in the NLC bulk in consequence of the easy axis spatial modulation and then it is tested by a plane light wave normally incident on the cell plane $z = L$ with the electric vector $\vec{E} = (0, E, 0)$ along the y axis. We assume that intensity of the incident light is sufficiently small and does not affect the initial distribution of director. Thus, the electric vectors of ordinary and extraordinary light waves appearing at the plane $z = L$ can be written as $E_o = E\cos\varphi_L(y)$, $E_e = E\sin\varphi_L(y)$, where $\varphi_L(y)$ is defined by expression (6). In the Mauguin limit [11], the amplitudes of the ordinary and extraordinary light waves passed through the cell take the form

$$E'_o = E\cos\varphi_L(y)\exp(ikLn_o),\ E'_e = E\sin\varphi_L(y)\exp(ikLn_e),$$

where n_e, n_o are extraordinary and ordinary refractive indexes, $k = 2\pi/\lambda$ is the wave vector of the incident light beam. The cartesian components read

$$E_x = E'_e\cos\varphi_0(y) - E'_o\sin\varphi_0(y),\ E_y = E'_e\sin\varphi_0(y) + E'_o\cos\varphi_0(y) \quad (10)$$

Diffraction pattern formed by the light field (10) can be found by taking the spatial Fourier transform of this field $\vec{\Phi}(\alpha) = \int_{-\infty}^{\infty} \vec{E}(y)\exp(-ik\sin\alpha y)dy$, where α is the angle of deviation of the diffracted light beam from the direction of the incident beam (along z axis). Since the light field (10) is a periodic function with period Λ, the diffracted field becomes [12]

$$\vec{\Phi}(\alpha) \sim \frac{\sin(1/2\,\Lambda Nk\sin\alpha)}{\sin(1/2\,\Lambda k\sin\alpha)}\int_{-\Lambda/2}^{\Lambda/2}\vec{E}(y,0)\exp(-ik\sin\alpha y)dy, \quad (11)$$

where N is the number of "gaps" which form the diffraction pattern. Substituting the field (10) in eq. (11) and taking into account that the condition for the m-th diffraction maximum is $m = (k/Q)\sin\alpha = (\Lambda/\lambda)\sin\alpha$, we obtain for the m-th order

$$\Phi_x^m \sim iJ_m(\Delta_1)\sin\frac{\chi}{2}\sin\left(\varphi_L+\varphi_0-\frac{\pi m}{2}\right)$$

$$+J_m(\Delta_2)\cos\frac{\chi}{2}\sin\left(\varphi_L-\varphi_0-\frac{\pi m}{2}\right) \qquad (12)$$

$$\Phi_y^m \sim iJ_m(\Delta_1)\sin\frac{\chi}{2}\cos\left(\varphi_L+\varphi_0-\frac{\Delta m}{2}\right)$$

$$+J_m(\Delta_2)\cos\frac{\chi}{2}\cos\left(\varphi_L-\varphi_0-\frac{\pi m}{2}\right). \qquad (13)$$

Here $J_m(x)$ is the Bessel function of the integer order m, $\Delta_1=\Delta\varphi_L+\Delta\varphi_0$, $\Delta_2=\Delta\varphi_L-\Delta\varphi_0$, $\chi=kL=2\pi(L/\lambda)(n_e-n_0)$.

As seen from the equations (12) and (13) the polarization state of the diffracted beam in the given NLC depends on the order of diffraction and the easy axis parameters.

Let's find the reduced Stokes parameters ξ_1, ξ_2, ξ_3 which characterize the degree of linear and circular polarization of light. They can be obtained from the Jones matrix $\hat{I}=\langle E_i E_j^*\rangle=(1/2)I_0(\hat{1}+\xi_1\hat{\sigma}_1+\xi_2\hat{\sigma}_2+\xi_3\hat{\sigma}_3)$, where $I_0=Sp\hat{I}$, $\hat{\vec{\sigma}}$ are the Pauli matrices, and have the form [13]

$$\xi_0=|\Phi_x|^2+|\Phi_y|^2,$$

$$\xi_1=2\mathrm{Re}(\Phi_x\Phi_y^*)/\xi_0,$$

$$\xi_2=-2\mathrm{Im}(\Phi_x\Phi_y^*)/\xi_0,$$

$$\xi_3=(|\Phi_x|^2-|\Phi_y|^2)/\xi_0. \qquad (14)$$

Substituting the expression (12), (13) we obtain

$$\xi_0=\sin^2(1/2\,\chi)\,J_m^2(\Delta_1)+\cos^2(1/2\,\chi)\,J_m^2(\Delta_2),$$

$$\xi_1=(-1)^{m+1}\{\sin^2(1/2\,\chi)\sin 2(\varphi_L+\varphi_0)\,J_m^2(\Delta_1)$$

$$-\cos^2(1/2\,\chi)\sin 2(\varphi_L-\varphi_0)\,J_m^2(\Delta_2)\}/\xi_0, \qquad (15)$$

$$\xi_2 = (-1)^{m+1} \sin\chi \sin(2\varphi_L) J_m(\Delta_1) J_m(\Delta_2)/\xi_0$$

$$\xi_3 = (-1)^{m+1} \{\sin^2(1/2\chi) \cos 2(\varphi_L + \varphi_0) J_m^2(\Delta_1)$$

$$+ \cos^2(1/2\chi) \cos 2(\varphi_L - \varphi_0) J_m^2(\Delta_2)\}/\xi_0.$$

Note that $\xi_1^2 + \xi_2^2 + \xi_3^2 = 1$, i.e. the light is perfectly polarized in all diffraction orders.

Let's consider the geometry usually realized in experiments, when the testing beam is incident on the cell plane with non-modulated easy axis ($\Delta\varphi_L = 0$). Then the expressions for the reduced Stokes parameters are simplified to

$$\xi_0 = J_m^2(\Delta\varphi_0),$$

$$\xi_1 = (-1)^{m+1} \{\sin^2(\chi/2)\sin 2(\varphi_L + \varphi_0)$$

$$- \cos^2(\chi/2)\sin 2(\varphi_L - \varphi_0)\},$$

$$\xi_2 = -\sin\chi \sin(2\varphi_L), \tag{16}$$

$$\xi_3 = (-1)^{m+1} \{\sin^2(\chi/2)\cos 2(\varphi_L + \varphi_0) + \cos^2(\chi/2)\cos 2(\varphi_L - \varphi_0)\}.$$

It is seen that only the sign of the reduced Stokes parameters ξ_2, ξ_3 changes from one diffraction maximum to the next one.

In the case of the equal directions of the easy axes on the cell boundaries ($\varphi_L = \varphi_0$) we obtain

$$\xi_1 = (-1)^{m+1} \sin^2(\chi/2) \sin 4\varphi_L,$$

$$\xi_2 = -\sin\chi \sin(2\varphi_L),$$

$$\xi_3 = (-1)^{m+1}\{1 - 2\sin^2(\chi/2)\sin^2 2\varphi_L\} \tag{17}$$

Thus, for the pure ordinary ($\varphi_L = \varphi_0 = 0$) or extraordinary ($\varphi_L = \varphi_0 = \pi/2$) light waves we have $\xi_1 = 0$, $\xi_2 = 0$, $\xi_3 = (-1)^{m+1}$, i.e. the diffracted beam is linearly polarized along the x or y axis in each diffraction order and adjacent diffraction orders have polarization vectors orthogonal to each other.

We can also find the diffraction efficiency of the grating defined as $\eta_m = \xi_0(m)/\xi_0(m=0)$. Under the small deviations of the easy axis

($\Delta\varphi_L, \Delta\varphi_0 \ll 1$) it has the form

$$\eta_m = \frac{\sin^2(\chi/2)(\Delta\varphi_L + \Delta\varphi_0)^{2m} + \cos^2(\chi/2)(\Delta\varphi_L - \Delta\varphi_0)^{2m}}{2^{2m}(m!)^2} \tag{18}$$

One can see that the diffraction efficiency decreases rapidly with the number of the diffraction order and does not depend on the period of the director grating.

In the case of not modulated easy axis on the cell boundary $z = L$ ($\Delta\varphi_L = 0$) the diffraction efficiency takes the form

$$\eta_m = J_m^2(\Delta\varphi_0)/J_0^2(\Delta\varphi_0) \tag{19}$$

and depends neither on the director grating period nor on the value of χ.

3.3. Influence of Finite Anchoring

For the sake of simiplicity let's consider the cell with non-modulated easy axis and the infinite director anchoring on the plane $z = L$ and modulated easy axis and the finite anchoring energy W^φ on the plane $z = 0$. In this case the unit vectors along the easy axis directions have the form

$$\vec{e}_L^{\,\varphi} = (1, 0, 0), \vec{e}_0^{\,\varphi} = (\cos\varphi_0(y), \sin\varphi_0(y), 0), \tag{20}$$

where $\varphi_0(y) = \Delta\varphi_0 \sin Qy$. Minimizing the free energy (1) in absence of the light field we obtain the equation for director $\vec{n} = (\cos\varphi, \sin\varphi, 0)$ and boundary conditions

$$\begin{cases} \Delta_{yz}\varphi = 0, \\ \varphi|_{z=L} = 0, \quad [L\partial\varphi/\partial z - (1/2)\xi^\varphi \sin 2\{\varphi - \varphi_0(y)\}]_{z=0} = 0, \end{cases} \tag{21}$$

where $\xi^\varphi = W^\varphi L/K$ is the azimuth anchoring parameter. In order to find the analytical solution to the boundary problem (21) let restrict outselves to small values of both the anchoring parameter ξ^φ and the easy axis deviation $\Delta\varphi_0$. In this case we can seek the solution as a power series in ξ^φ. As a result one can obtain the approximate solution in the form

$$\varphi = \Delta\varphi_0\, \xi^\varphi \frac{\sinh Q(L-z)}{QL\cosh QL} \sin Qy \tag{22}$$

Comapring this result with the formula (7) for the case $\varphi|_{z=L}=0$ one can see that in the cell with weak anchoring the deviation angle $\Delta\varphi_0$ is replaced by the re-normalized one $\Delta\tilde{\varphi}_0=\Delta\varphi_0\,\xi^{\varphi}(\tanh QL)/QL$. Since for $\Delta\varphi_0 \ll 1$ the diffraction efficiency of the planar grating is proportional to the squared easy axis deviation angle (see Exp. (18)), in the case of the weak anchoring we obtain

$$\eta_\xi = \eta_{\xi=\infty}\left(\xi^{\varphi}\frac{\tanh QL}{QL}\right)^2 \tag{23}$$

Thus, the diffraction efficiency of the planar cell depends now on the period of the easy axis modulation (Fig. 2). At the same time, the polarization state of the diffracted beams does not depend on the amplitude of the easy axis modulation (see expression (17)), and hence does not change.

4. PERIODIC SPATIAL MODULATION OF HOMEOTROPIC EASY AXIS

4.1. Diffraction on the Nearly Homeotropic Director Grating

Let consider the director grating appearing in the cell bulk only due to the spatial modulation of the easy axes near the homeotropic direction. In this

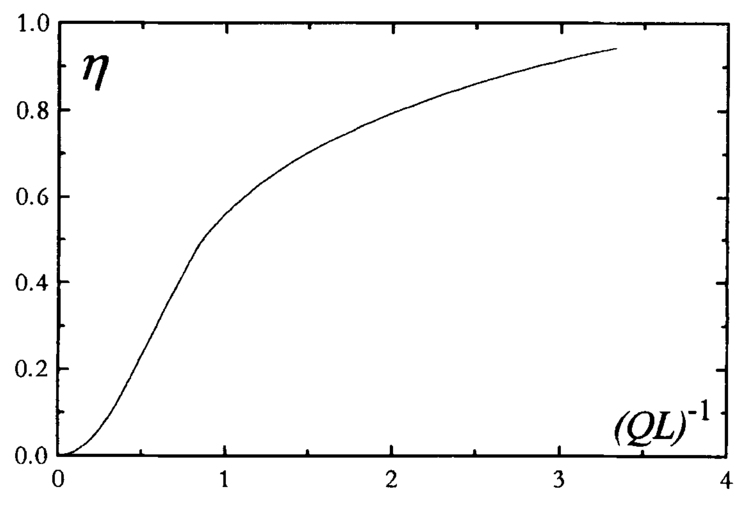

FIGURE 2 Dependence of the diffraction efficiency of the planar grating on the spatial period of easy axis modulation in the case of small anchoring energy.

case the equation for director and the boundary conditions are similar to the ones obtained in the planar case. Thus, the director distribution is given by an expression which is analogous to the expression (7) where it is necessary to replace φ^0, $\varphi_{0,L}$, $\Delta\varphi_{0,L}$ by θ^0, $\theta_{0,L}$, $\Delta\theta_{0,L}$ respectively. Further we put $\theta_{0,L} = 0$, $\Delta\theta_{0,L} \ll 1$.

Let an extraordinary plane light wave with polarization vector along the y axis be normally incident on the cell plane $z = L$ and be diffracted by this grating. We assume that the intensity of the light wave is small and does not affect the initial director distribution. Thus the light field in the cell in the approximation of geometrical optics and for large periods of the easy axis modulation can be written as [10]

$$\vec{E}\,(y, z) = (0, E_y, E_z), \tag{24}$$

where

$$E_y = A(\varepsilon_\perp + \varepsilon_a n_z^2)^{1/4} \exp\left[i\frac{\omega}{c}\Psi(y) \right],$$

$$E_z = \frac{-A\,\varepsilon_a n_y n_z}{(\varepsilon_\perp + \varepsilon_a n_z^2)^{3/4}} \exp\left[i\frac{\omega}{c}\Psi(y) \right],$$

$$\Psi(y) = \int_0^L \sqrt{\frac{\varepsilon_\parallel \varepsilon_\perp}{(\varepsilon_\perp + \varepsilon_a n_z^2)}}\, dz.$$

Here $n_z(y, z) = \cos\theta(y, z)$ and we took into account that for $Q\lambda \ll 1$ the light field depends on the y coordinate as parameter. Thus, both the amplitude and the phase of the light field behind the cell are modulated with period $\Lambda/2 = \pi/Q$. Expanding the light field (24) as a power series in $\varepsilon_a(\Delta\theta)^2 \ll 1$ we obtain the next approximate expression for the amplitude $\Phi(\alpha)$ of the diffracted light beam of the m-th order of diffraction

$$\Phi(\alpha) \sim (1 + \gamma) J_m(\delta) + (1/2)\, i\gamma\, [J_{m+1}(\delta) - J_{m-1}(\delta)] \tag{25}$$

where

$$\delta = \pi/8\sqrt{\varepsilon_\perp}\frac{\varepsilon_a}{\varepsilon_\parallel}\frac{L}{\lambda} \times$$

$$\times \frac{(\Delta\theta_L^2 + \Delta\theta_0^2)(\sinh(2t) - 2t) + 4\Delta\theta_L\,\Delta\theta_L\,\Delta\theta_0\,(t\cosh t - \sinh t)}{t\sinh^2 t}, \quad t = QL,$$

$\gamma = (1/8)(\varepsilon_a/\varepsilon_\parallel)(\Delta\theta_L^2 - \Delta\theta_0^2)$. Here we take into account, that the condition for the m-th order of diffraction is $m = (\pi/\lambda Q)\sin\alpha$.

The light intensity in the m-th order of diffraction up to terms $\sim(\Delta\theta_{0,L})^2$ has the form

$$I_m \sim |\Phi(\alpha)|^2 = (1 + 2\gamma)\, J_m^2(\delta)$$

Thus the diffraction efficiency of the grating is $\eta_m = I_m/I_0 = J_m^2(\delta)/J_0^2(\delta)$. Under the small deviations of easy axes and the symmetric boundary conditions $(\Delta\theta_0 = \Delta\theta_L = \Delta\theta)$ it has the form

$$\eta_1^{\mathrm{sym}} = \left[\frac{\pi(\Delta\theta)^2}{8}\sqrt{\varepsilon_c}\,\frac{\varepsilon_a}{\varepsilon_\parallel}\,\frac{L}{\lambda}\,\frac{QL + \sinh(QL)}{QL\cosh^2(QL/2)}\right]^2 \tag{26}$$

For the cell with modulated easy axis only on the plane $z = 0$ $(\Delta\theta_L = 0)$ this dependence is given by

$$\eta_1^{\mathrm{single}} = \left[\frac{\pi(\Delta\theta_0)^2}{16}\sqrt{\varepsilon_\perp}\,\frac{\varepsilon_a}{\varepsilon_\parallel}\,\frac{L}{\lambda}\,\frac{\sinh(2QL) - QL}{QL\sinh^2(QL)}\right]^2 \tag{27}$$

One can see that the diffraction efficiency is proportional to the forth power of the easy axis deviation angle unlike the quadratic dependence in the case of the planar grating. The dependence of the diffraction efficiencies (26) and (27) on the dimensionless period of the easy axis modulation $(QL)^{-1}$ is shown in Figure 3.

4.2. Influence of Finite Anchoring

As it was mentioned above, in the one-elastic-constant approximation the equations for director and boundary conditions are similar to ones used in the planar case. Let's consider as in Sec 2.3 the cell with non-modulated easy axis and infinite anchoring on the plane $z = L$ and the modulated one with finite anchoring energy W^θ on the opposite plane $z = 0$. Introducing the director as $\vec{n} = (0, \sin\theta, \cos\theta)$ and the unit vector of easy axis on the plane $z = 0$ as $\vec{e}^{\,0} = (0, \sin\theta_0, \cos\theta_0)$, where $\theta_0 = \Delta\theta_0 \sin(Qy)$, we obtain in case of small anchoring parameter $\xi^\theta = W^\theta L/K \ll 1$ and $\Delta\theta_0 \ll 1$ the expression for the director distribution similar to that one of the planar cell

$$\theta = \Delta\theta_0\, \xi^\theta \frac{\sinh Q(L - z)}{QL\cosh QL} \sin Qy \tag{28}$$

Since the diffraction efficiency in the homeotropic cell is proportional to the forth power of the easy axis modulation angle (see expression (27)), under the weak anchoring we obtain

$$\eta_\xi = \eta_{\xi=\infty}\left(\xi^\theta \frac{\tanh QL}{QL}\right)^4 \tag{29}$$

The dependence of the diffraction efficiency (29) on the dimensionless period of the easy axis modulation is presented in Figure 3.

5. CONCLUSIONS

We have studied the influence of the spatial modulation of the easy axis direction onto the cell surfaces on the director distribution in the NLC cell bulk reoriented by a light field with spatially modulated intensity. Both planar and homeotropic director configurations were considered. The obtained director distributions show that the director gratings with periods $2\pi/Q$ of easy axis modulation, $2\pi/\Delta q$ of the light intensity modulation and $2\pi/Q + \Delta q$, $2\pi/Q - \Delta q$ appears in the cell bulk.

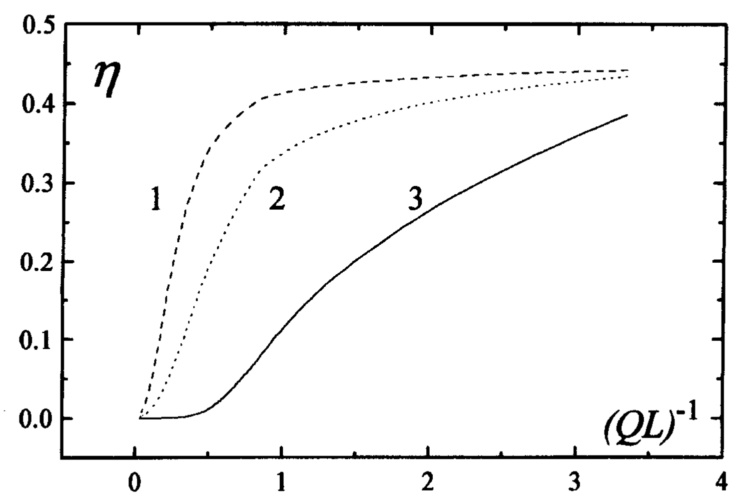

FIGURE 3 Diffraction efficiency of the nearly homeotropic grating as a function of the easy axis modulation period: 1) symmetric modulation on both cell planes; 2) modulation only on a single plane; 3) modulation on a single plane with small anchoring parameter on this plane.

Diffraction of a probe light beam by the director gratings caused only by the easy axis modulation have been considered as well. In the planar cell the diffracted beam changes its polarization from one order of diffraction to the next. For the particular geometry, when only ordinary or extraordinary light wave is incident on the plane with non-modulated easy axis, the diffracted beam is linearly polarized, and adjacent diffraction orders have polarization vectors orthogonal to each other. The diffraction efficiency of the planar grating does not depend on the period of easy axis modulation and is proportional to the squared amplitude of modulation.

In the nearly homeotropic cell both the amplitude and phase of the probe light beam changes in the cell bulk. In the geometrical optics approximation diffraction efficiency of this director grating, depends on the period of easy axis modulation and is proportional to the fourth power of the amplitude of modulation. Weak anchoring on the cell boundaries does not change the polarization state of the diffracted beam, modifying only the dependence of the diffraction efficiency on the period of easy axis modulation.

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