Two-Dimensional Collapsing Bond Animals

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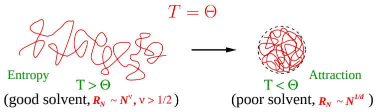
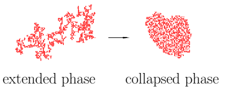
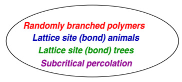
Collapse Transitions

Unbranched polymers: coil-globule transition at $T = \Theta$



Branched polymers:





Same universality class

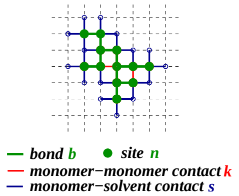
Interacting Lattice Animals

Partition sum: $Z_N(y,\tau) = \sum_{b,k} C_{Nbk}\, y^{b-N+1}\tau^k$,
C_{Nbk}: number of configs., y and τ: fugacities
$4N = 2b+2k+s$, s: number of monomer-solvent contacts

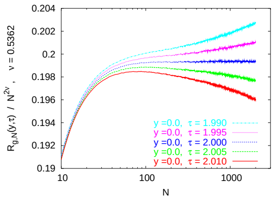
- **Unweighted animals:** $y = \tau = 1$
- **Bond percolation:** $y = p/(1-p)^2,\ \tau = 1/(1-p),\ 0 \le p \le 1$
  Critical percolation point: $y = 2,\ \tau = 2$, as $p = p_c = 1/2$
- **Collapsing trees:** $y = 0\ (b = N-1)$
- **Strongly embeddable animals:** $\tau = 0\ (k = 0)$

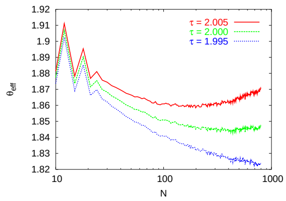
Scaling laws:

- $Z_N \sim \mu^{-N} N^{-\theta}$
- $R_N \sim N^{\nu}$



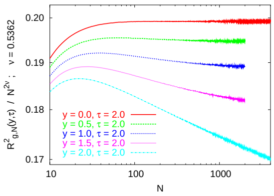
Collapsing Trees: $y = 0$

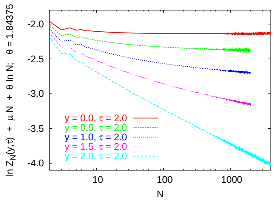




Cross-Over: $(0 \le y \le 2)$

critical percolation → collapsing trees



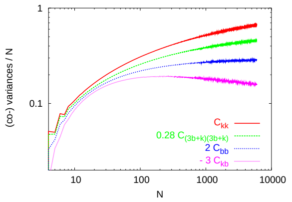


Algorithm

A sequential sampling and depth-first method with resampling based on the pruned-enriched Rosenbluth method (PERM)

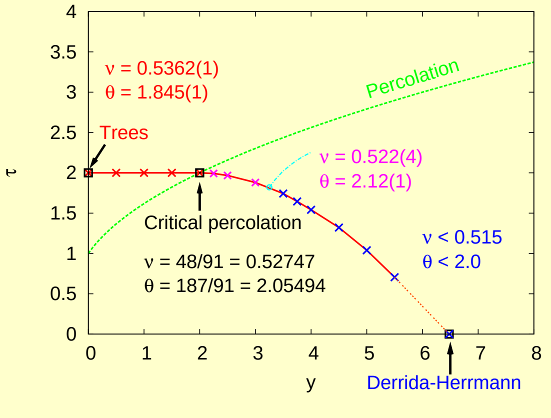
Bond Percolation Point $(y = 2,\ \tau = 2,\ p_c = 1/2)$

- **Partition sum:** $Z_N^{\rm perc} = \sum_{b,k} C_{Nbk}\, p^b (1-p)^{k+s}$
- **Scaling ansatz near** $p = p_c$: $z_N^{\rm perc}(p) \approx N^{-5/91} F((p-1/2)N^{\sigma})$, $\sigma < 1/2$
- **Scaling laws at** $p = p_c$: $z_N^{\rm perc}(p_c) \sim N^{-5/91}$, $R_N^2 \sim N^{96/91}$



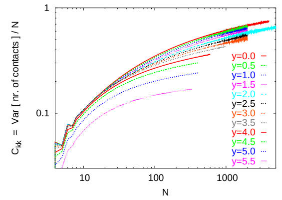
- $< 3b + k >= 4N$,
  ${\rm var}[3b+k] = 2 < b > + O(N^{2\sigma})$
- **(Co-)variances divided by** N:
  $C_{ij} = (< ij > - < i >< j >)/N$
- **If** ϕ **defined by** $C_{kk},\ C_{bb} \sim N^{2\phi-1}$
  $\Rightarrow \phi = 1/2$ (not $\phi = \sigma$)
  holds for entire transition line

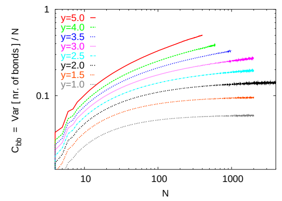
Phase Diagram for Interacting Animals



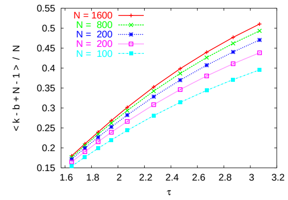
- **Different universality classes on the transition curve for:**
  - **Collapsing trees,** $y = 0,\ \tau = 2$
  - **Critical percolation,** $y = 2,\ \tau = 2$
  - **Intermediate region,** $2 < y \le 3.2$
  - **Derrida-Hermann,** $y \approx 6.48,\ \tau = 0$
- **Two different collapsed phases:**
  contact-driven, bond-driven

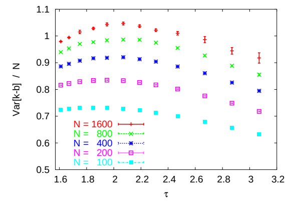
Region: $0 \le y \le 5.5$





Collapsed phases: $y = 3.75$





Reference

1. Hsu, Nadler and Grassberger J. Phys. A: Math. Gen 38, 775 (2005)
2. Hsu and Grassberger e-print cond-mat/0504678, J. Stat. Mech., in press (2005).