John von Neumann - Institut für Computing

Simulations of Lattice Animals

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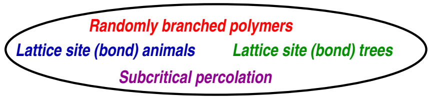
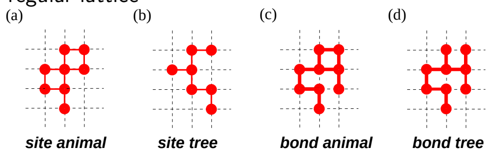
Models

1. Same universality class



2. Definitions: Clusters of connected sites (bonds) on a regular lattice



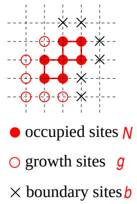
Features of the Animal Problem

1. Animals in the bulk
   - Upper critical dimension: $d = 8$
   - Exactly known ν and θ in $d = 3,\ 4$ and ≥ 8
   - No exact value of exponent ν in $d = 2$
   - Parisi-Sourlas prediction: $\theta = (d-2)\nu + 1$
2. Animals with one site attached to an attractive surface
   - For weak attraction (high temperature), $Z'_N \sim \mu'^N N^{-\theta'}$ with $\mu' = \mu$ and $\theta' = \theta$
   - For strong attraction (low temperature), phase transition to adsorbed phase cross-over exponent $\phi = 1/2$ for $d > 2$

Algorithm

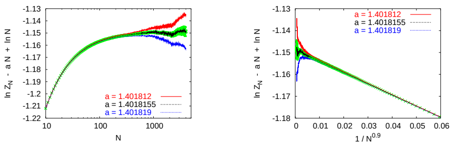
A novel Monte Carlo algorithm based on the pruned-enriched Rosenbluth method (PERM)

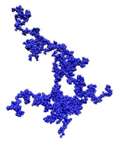
- Start with percolation cluster growth algorithm
- Animal partition sum: $Z_N = p^{-N}\left\langle (1-p)^{-b} \right\rangle$ (sample average over all growing clusters)
- Criteria for cloning and pruning: fitness function:
  $$f_n = w_n/(1-p)^g = p^{-n}(1-p)^{-b-g}$$
  If $f_n > c_+(f_n)$ cloning
  If $f_n < c_-(f_n)$ pruning
- Optimal values of p : $p < p_c$ and $p \to p_c$ for $N \to \infty$



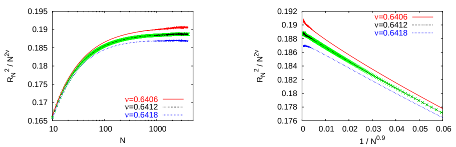
Animals in the Bulk

Partition sum: $Z_N \sim \mu^N N^{-\theta}(1 + b_Z N^{-\Delta} + \cdots)$

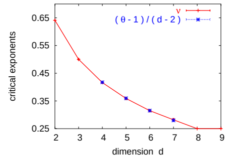




Radius of gyration: $R_N^2 \sim N^{2\nu}(1 + b_R N^{-\Delta} + \cdots)$

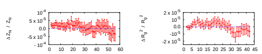


Parisi-Sourlas prediction ✓



Relative errors:

I. Jensen (J. Stat. Phys. 102, 865 (2001)



References:

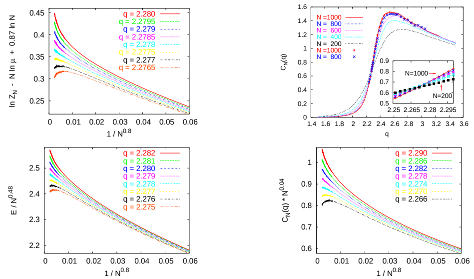
cond-mat/0408061, cond-mat/0411262

Animals Attached to an Attractive Surface

Partition sum: $Z_N^{(1)}(q) \sim \mu^N N^{-\theta_s}\Psi[(q-q_c)N^{\phi}]$

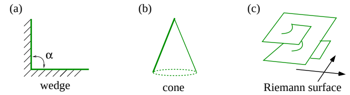
Energy: $E_N(q_c) \sim N^{\phi}(1 + b_E N^{-\Delta_E} + \cdots)$

Specific heat: $C_N(q) \sim (q-q_c)^{-\alpha(=1-1/\phi)}$
$\sim N^{2\phi-1}$ at $q = q_c$



$q_c = 2.2778(8)$, $\phi = 0.480(4)$, for $d = 2$
$\phi = 0.50(1),\ 0.50(2),\ 0.51(3)$, for $d = 3,\ 4,\ 5$

2-d Clusters Grafted to



Conformally invariant models:

SAW: $\gamma \propto \frac{1}{\alpha}$

Critical percolation: $\tau - 2 \propto \frac{1}{\alpha}$

But : Animals : $\theta \sim \begin{cases} 1/\alpha & \alpha \to 0 \\ -\alpha/2\pi & \alpha \to \infty \end{cases}$