

# General 2-Fluid Hydrodynamics of Complex Fluids

H. Pleiner<sup>†</sup> and J.L. Harden<sup>‡</sup>

<sup>†</sup>*Max-Planck-Institute for Polymer Research, 55021 Mainz, Germany*

<sup>‡</sup>*Chemical Engineering, Johns Hopkins University, Baltimore, MD 21218, USA*

## Abstract

We discuss general 2-fluid hydrodynamic equations for complex fluids, where one kind is a simple Newtonian fluid, while the other is polymeric/elastomeric, thus being applicable to polymer solutions and swollen elastomers. The procedure can easily be generalized to other complex fluid solutions. Special emphasis is laid on such nonlinearities that originate from the 2-fluid description, like the transport part of the total time derivatives. It is shown that the proper velocities, with which the hydrodynamic quantities are convected, cannot be chosen at will, since there are subtle relations among them. Within allowed combinations the convective velocities are generally non-universal, i.e. material dependent. The so-called stress division problem, that means how the elastic stresses are distributed between the two fluids, is shown to depend partially on the choice of the convected velocities, but is otherwise also material dependent. A set of reasonably simplified equations is given as well as a linearized version of an effective concentration dynamics that may be used for comparison with experiments.

## Introduction

The thermodynamic and hydrodynamic properties of multi-component complex fluids are determined by the microscopic degrees of freedom of their constituents and the coupling between these degrees of freedom. Such systems can exhibit rather rich phase behavior and dynamics, especially when one or more components is a structured or macromolecular fluid. Due in part to the coupling of internal degrees of freedom, these systems can also exhibit novel flow-induced structural evolution phenomena, including shear-induced phase transformations and flow alignment of constituents on microscopic to mesoscopic length scales. Such structural evolution in turn leads to nonlinear rheological behavior, such as stress overshoots in response to imposed rates of strain, plasticity, and thixotropy.

The overwhelming complexity of the microscopic description of these systems, such a detailed description is often not well suited for analysis of the macroscopic dynamical behavior. Instead, explicit macroscopic models have been developed for this purpose. Some such models have been obtained by a suitable coarse-graining procedure starting

from a microscopic theory. Others are purely phenomenological models constrained only by conservation laws, symmetry considerations and thermodynamics. The so-called “two-fluid” models for binary systems of distinct components or phases are useful examples of such a macroscopic approach. In the two-fluid description, each component or phase is treated as a continuum described by local thermodynamic variables (e.g. temperature, density, and relevant order parameters), and dynamical quantities (e.g. velocity or momentum). In general, these variables for the constituents are coupled. For instance, the effective friction between components in a binary fluid mixture leads to a drag force in the macroscopic description that is proportional to the local velocity difference.

Two-fluid models have been employed in many different physical contexts. The two-fluid approach is a key element of many traditional models for multi-phase flow of bubbly liquids, fluid suspensions of particulates, and binary mixtures of simple fluids. Other examples in condensed matter physics include two-fluid models for superfluid helium, dynamics of plasmas, transport in superconductors, viscoelasticity of concentrated fluid emulsions, flow-induced ordering of wormlike micelle solutions, flow of colloidal suspensions. Two-fluid models have been used extensively to model a wide range of dynamical phenomena in polymer solutions and binary blends, including the hydrodynamics modes of quiescent polymer solutions, kinetics of polymer dissolution, hydrodynamics and rheology of polymer solutions and blends, and polymer migration and phase separation under flow.

These examples share certain general features. In each, two distinct species or coexisting phases (gas and liquid, normal fluid and superfluid, polymer and solvent, mesogens and solvent etc.) with mass densities  $\rho_1$  and  $\rho_2$ , which are conserved individually in the absence of chemical reactions, move with distinct velocities  $\mathbf{v}_1$  and  $\mathbf{v}_2$ , respectively. Due to (usually strong) internal friction, the momenta of the constituent species,  $\rho_1\mathbf{v}_1$  and  $\rho_2\mathbf{v}_2$ , are not conserved individually. Of course, total momentum is conserved. In most cases of fluid mixtures the friction is so strong that the velocity difference  $\mathbf{v}_1 - \mathbf{v}_2$  is nonzero for very short times only, i.e. it is a very rapidly relaxing quantity that is not included in the hydrodynamic description for binary mixtures. However, there are systems and situations, where the relaxation of the relative momenta is slow enough to have a significant influence even on the hydrodynamic time scale. Then a two-fluid description is appropriate and useful.

In this talk we focus on a general nonlinear two-fluid description of complex fluids, where one species is a viscous Newtonian fluid and the other is a polymer. Emphasis is placed on the rigorous derivation of the dynamic equations within the framework of hydrodynamics as contrasted to ad-hoc treatments. The resulting equations are rather general and complicated. They can and have to be simplified for special applications or systems by appropriate and well-defined approximations. One of the advantages of starting from the general theory is the possibility to identify and characterize the approximations made. The hydrodynamic method is quite general and rigorous, being based on symmetries, conservation laws, and thermodynamics.