

COMMENT ON  
"COUPLING BETWEEN ELASTIC DEFORMATION AND CONCENTRATION  
IN A TWO-COMPONENT NEMATIC LIQUID CRYSTAL"

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Recently [1] the existence of a cubic cross-coupling term between concentration variations and splay of the director field was pointed out and it was argued that this contribution leads a) to a reduction of the energy associated with dislocations and b) to a stabilization of high-strength disclinations in doped nematics.

Here we show a) that quartic terms in the energy are needed to preserve thermostatic stability; and that the inclusion of these quartic terms (in addition to the suggested cubic one) leads b) to an *increase* in the energy associated with dislocations and c) to a *destabilization* of high-strength disclinations in doped nematics. As a result there is still no mechanism known to date, that would be compatible with the stationary existence of high-strength disclinations.

Motivated by the observation of high strength dislocations in doped nematics on a fairly long time scale [2], Ragunathan tried to explain their enhanced stability in these mixtures. Such long lived, high-strength defects in nematic liquid crystals have also been observed in lyotropic low molecular weight systems [3] as well as in polymeric liquid crystals [4,5]. Thus the need for an explanation of the long, and - perhaps - infinite, lifetime appears well documented. All these systems have in common that they are not single phase (case of the low molecular weight systems) or not monodisperse (the polymeric systems). Therefore it seems quite natural to consider a coupling of director deformations to concentration variations as a way to resolve the puzzling experimental results [1].

Accordingly Ragunathan suggests a cubic contribution to the energy, which is quadratic in the splay field and linear in the concentration variations. Using only this term in addition to the terms classically present for the deformation energy of nematic liquid crystals, Ragunathan then calculated the energy of dislocations and of high-strength defects. He concluded that this one additional contribution to the energy is enough to explain the experimental observations [2].

However, the energy written down in [1] is not a positive definite form as it must be

to guarantee local thermostatic stability [6,7]. If the energy were *not* bounded from below, the system were unstable and small fluctuations would drive it into a different ground state. However, expanding the energy around this new (stable) ground state would result in a form, whose positivity were required by thermostatic stability. Thus, to guarantee stability, quartic terms must be added to the energy suggested in [1]. Using the notation of [8] and [1] whenever possible, we have for the energy the expression (keeping all terms up to quartic order relevant for the discussion below)

$$f = \frac{1}{2} \int \int ([K_1 - \alpha\delta c + \eta(\delta c)^2](div\hat{\mathbf{n}})^2 + \beta(\delta c)^2 + \zeta(\delta c)^3 + \mu(\delta c)^4 + \gamma(div\hat{\mathbf{n}})^4) dx dy \quad (1)$$

For eq.(1) the condition of thermostatic stability leads to the positivity conditions

$$K_1 \geq 0, \quad \gamma \geq 0, \quad \beta \geq 0, \quad \mu \geq 0, \quad \eta \geq 0 \quad (2a)$$

$$4(\beta\gamma + K_1\eta) - \alpha^2 \geq 0 \quad (2b)$$

$$4\beta\mu - \zeta^2 \geq 0 \quad (2c)$$

For the same cylindrical geometry as in ref.[1] we obtain instead of eq.(7) in [1] taking into account the effects of the quartic terms  $\sim \gamma$  and  $\sim \eta$

$$f \approx \pi K_1 \left[ \ln \left( \frac{\rho_2}{\rho_1} \right) - \frac{\eta}{2\beta\rho_1^2} \right] + \frac{\pi}{2} \left( \gamma + \frac{K_1\eta}{\beta} - \frac{\alpha^2}{4\beta} \right) \frac{1}{\rho_1^2} + \pi \frac{\alpha^2}{8\eta} \left[ \frac{\eta}{\beta\rho_1^2} - \ln \left( 1 + \frac{\eta}{\beta\rho_1^2} \right) \right] \quad (3)$$

where  $\rho_1$  is the radius of the inner cylinder and  $\rho_2$  that of the outer one. Similarly we find for disclination defects of strength  $s$ , instead of eq.(12) in ref.[1]

$$\begin{aligned} \frac{f}{\pi s^2} \approx & K \left[ \ln \left( \frac{L}{\rho_c} \right) - \frac{\eta s^2}{2\beta\rho_c^2(s)} \right] + \left( \gamma + \frac{K\eta}{\beta} - \frac{\alpha^2}{4\beta} \right) \frac{s^2}{2\rho_c^2(s)} \\ & + \frac{\alpha^2}{8\eta} \left[ \frac{\eta s^2}{\beta\rho_c^2(s)} - \ln \left( 1 + \frac{\eta s^2}{\beta\rho_c^2(s)} \right) \right] \end{aligned} \quad (4)$$

where  $\rho_c(s)$  is the core radius,  $s$  the defect strength and  $L$  the radius of a cylindrical region [1]. There is neither a lowering of a defect energy nor a stabilization of high-strength

defects, since the second (via (2b)) and third contribution in (4) are manifestly positive and corrections of the order  $L^0$  to the main result [9]  $\ln(L/\rho_c)$  are generally neglected (note that  $\eta/\beta \leq \rho_c^2$  for a gradient expansion like (1)). All contributions scale like  $s^2$ , since  $\rho_c(s) \sim s$  [10], and  $f$  scales globally with  $s^2$  (apart from logarithmic corrections). Thus, none of the various energy contributions will change its importance relative to others by considering higher defect strengths  $s$ . Therefore on the basis of eq.(4) a decay of high-strength defects, such as for example  $s = \pm 3/2$  and  $s = \pm 2$ , into defects of lower strength is expected.

The remaining terms  $\zeta(\delta c)^3$  and  $\mu(\delta c)^4$  in eq.(1) do not alter these results due to the boundedness of  $\zeta$  (eq.(2c)), although a simple analytical expression can no longer be given in this most general case. We close by noting that similar cubic terms as in [1] have been considered before in [11] in a different context.

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